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Lectures 13 and 14 covered sampling. Here are some practice problems for review.

Problem 1 (Nyquist frequency) Can the following signals be reconstructed with sinc interpolation? If so, what is the minimum frequency ω_s at which the signal should be sampled?

(a) $x(t) = \cos(2\pi t) + \sin(\pi t)$

(b) $x(t) = \text{rect}(t)$

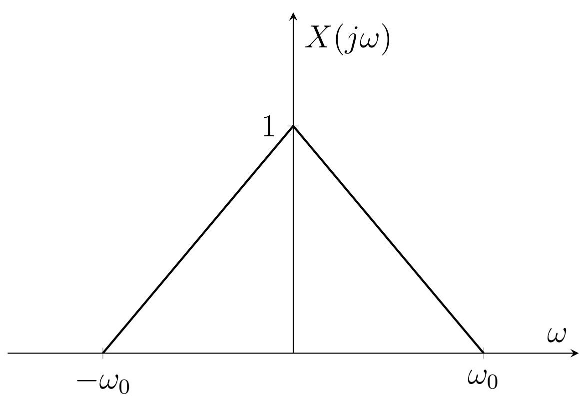
(c) $x(t) = \text{sinc}(t)$

(d) $x(t) = \text{sinc}(t) * \text{sinc}(t)$

(e) $x(t) = \text{sinc}^2(t)$

(f) $x(t) = e^{-t^2}$

Problem 2 (Aliasing) A real analog signal $x(t)$ has CTFT $X(j\omega)$:



We define the impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

for sampling rate T.

(a) Let $\omega_N = \frac{2\pi}{T_N}$ be the Nyquist frequency corresponding to a signal with the bandwidth of $x(t)$ above. What is T_N? Sketch $P(j\omega)$, the CTFT of $p(t)$, when $T = T_N$.

(b) Sketch $X_N(j\omega)$, the CTFT of $x_N(t) = x(t)p(t)$, when $T = T_N$.

(c) Let $x_A(t)$ be $x(t)$ sampled at frequency $\omega_s = 2\omega_N$. Sketch $X_A(j\omega)$.

(d) Let $x_B(t)$ be $x(t)$ sampled at frequency $\omega_s = \frac{3\omega_0}{2}$. Sketch $X_B(j\omega)$. Indicate the region where aliasing occurs.

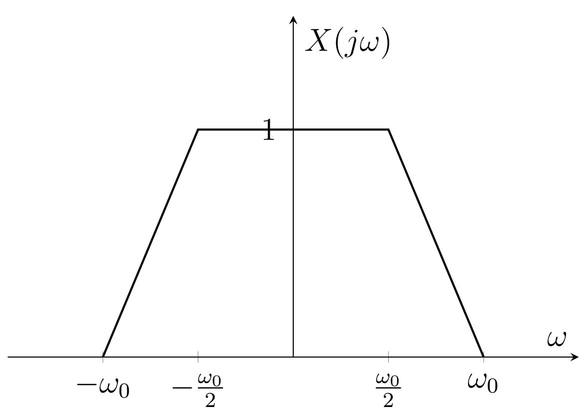
(e) Sketch $H(j\omega)$, the the CTFT of $h(t)$, the ideal reconstruction filter for a signal with the same bandwidth as $x(t)$.

(f) Draw $X_{BR}(j\omega)$, the CTFT of $x_{BR}(t) = x_B(t) * h(t)$.

(g) Suppose that instead of the spectrum given above, we had $x(t) = \cos(\omega_0 t)$. Now what is $X_{BR}(j\omega)$?

(h) For $x(t) = \cos(\omega_0 t)$, draw $x(t)$ and $x_{BR}(t)$ on the same plot. Circle the points where $x(t)$ was sampled to generate $x_B(t)$.

Problem 3 (Downsampling and Upsampling) Consider a signal $x(t)$ with the following spectrum:



Let $x_A[n]$ be the vector of samples taken from $x(t)$ with frequency $\omega_A = 4\omega_0$.

(a) Sketch $X_A(e^{j\Omega})$, the DTFT of $x_A[n]$.

(b) Let $x_B[n] = x_A[2n]$. How would you obtain $x_B[n]$ by sampling $x(t)$, i.e., to what sampling rate T does $x_B[n]$ correspond? Sketch $X_B(e^{j\Omega})$. Is there aliasing?

(c) Let $x_C[n] = x_A[3n]$. What is T for this sampled signal relative to $x(t)$? Sketch $X_C(e^{j\Omega})$. Is there aliasing?

(d) Let $x_D[2n] = x_A[n]$ and $x_D[2n+1] = 0$. Devise a scheme to recover $x(t)$ from $x_D[n]$ *without* downsampling. Is there aliasing?

Problem 4 (Uniform Sampling) This problem was inspired by 5.10 from Osgood (2019), *Lectures on the Fourier Transform and Its Applications*.

Suppose you are sampling a real signal $x(t)$ with the spectrum as given in Problem 2 above for $\omega_0 = \pi$. You take evenly spaced samples, but they are not necessarily centered at zero. Match the impulse trains $p(t)$ used for sampling to the resulting spectra of the sampled signal $x_S(t) = x(t)p(t)$.